M. Math. I year / 2005 – 2006 / Fourier Analysis

Mid-semester exam

Answer as many questions as you can. Time: 2 hrs.

1. f is a continuous 2π-periodic function whose Fourier coefficients are given by $\hat{f}(n) = \frac{1}{|n|^3+1}$. Prove that $S_n(f)(x) \xrightarrow{u} f(x)$. Find a reasonable value of n s.t. $\forall N \geq n$, $|S_N(f)(x) - f(x)| \leq \frac{1}{100}$.

2. Compute the F.T. of $\frac{1}{x^2+2x+5}$ and $\frac{2x+1}{x^2+2x+5}$.

   (Hint: First compute the F.T. of $e^{|x|}$.)

3. Let f be in the Schwartz space. Let $g_N(x)$ be defined by: $g_N(x) = \frac{1}{2\pi}\int_{-N}^{N} \hat{f}(\lambda) e^{i\lambda x}$. ~~Justify~~

   Prove that $\int_{-N}^{N} |f(x) - g_N(x)|^2 \to 0$ as $N \to \infty$.

   ~~You~~ You may quote Theorems proved in class.

4. Let $f(x) = \chi_{[-1,1]}(x)$. Compute explicitely $(f*f)(t)$, and then calculate $(f*f)^\wedge(\lambda)$. Is there an easier way to do ~~this~~ the second part?

5. Let $|f(x)| \leq e^{-|x|}$. Prove that if f is a non-trivial function, then $\nexists$ $m\{y : \hat{f}(y) \neq 0\}$ has to be infinite.

   (Hint: Can you find an open set $\Omega \subseteq \mathbb{C}$ s.t Ω contains the real line and $\hat{f}$ extends to a holomorphic function on Ω?)